**Thermal Equilibrium Properties**

Let’s calculate some thermodynamic properties. There are a lot of thermodynamic properties that can be related to the interaction parameters Fa,s0,1. First have to establish the quasi-particle distribution function.

**Fermi distribution function for quasi-particles**

So I’ll start by saying that as we go from free electrons to interacting electrons, I don’t think that the entropy changes, for the same reason that it wouldn’t if we gradually turned on an external field – ‘cause adiabatic work doesn’t cause entropy to change. And since it is given by (see Stat Mech/Independent Particles files):



in the free state (where npσ is the Fermi occupation number), it should be in the interacting state as well. *But* npσ would still be given in terms of the free particle energy eigenvalues p2/2m, and the free particle chemical potential. We want it, rather, in terms of the interacting energy (quasi-particle energy εpσ) and chemical potential. So gotta do a little more work. What we’ll do is plug this expression, as well as the expression for the number of particles:



(since quasi-particles are in 1-1 correspondance with particles, the number of particles and quasi-particles is the same) into the formula dE = TdS + μdN, where:



So to get the differentials we’ll say that we start from the GS, and add a free particles/quasi-particles to it. Then we have:



and,



and,



but well, both terms in δE are of the same order really, as discussed in the interaction file. So I think we do:



Now putting these all together into the differential relationship,



and equating coefficients of the arbitrary δnpσ, we have:



So we have the usual Fermion thermal occupation number formula:



**Heat Capacity**

Let’s start with the heat capacity. I’ll presume we’re in an isotropic system, say. And then we know the energy levels are given by (see previous file)



But this won’t play a role really. With the energies, we could get cV by proceeding straight from the definition cV = ∂u/∂T, but this causes some problems – I don’t know how to get the result we want w/o pressuming to know more about the density of states than just its value at the Fermi surface. It’s easier to start with cV = T∂s/∂T. So let’s start with the entropy.



From the previous file, we know that since we’re in an isotropic situation, the energy spectrum εpσ = εpσ(0). But whatever. If we introduce the interacting density of states, we can write this more succinctly.



We won’t, ultimately, presume to know what ρ is except at the Fermi surface. Now take the derivative of S w/r to T.



Well,



So now we have:



Now let’s use the chain rule:



So,



We can extend the lower bound of the integral to -∞ in the T → 0 limit, as ε0 – μ is negative and β → ∞. Furthermore, ∂/∂ε [1/(eε+1)] is narrowly peaked about ε = 0, and so it is legitimate to expand ρ about μ to first order in ε/β. Well, in fact we’ll just keep the zeroth term. μ is a temperature-dependent term as well, but it goes to εF in the T → 0 limit. So altogether,



That integral is -π2/3. So now we have:



And then the heat capacity follows from cV = T∂s/∂T.



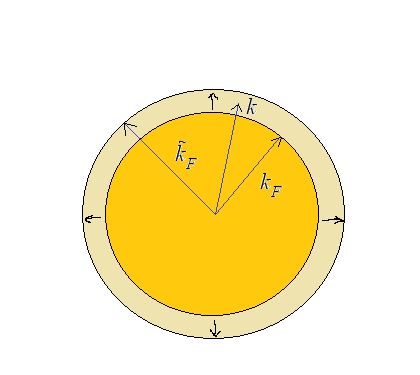
So we never assumed anything about the density of states except it’s behavior in the ground state, near the Fermi-surface, which we know. And we find a constant specific heat with renormalized mass.

**Compressibility**

Can start with this formula for the isothermal compressibility (the first equality is a definition, and the latter equality was shown to be true, at least near T = 0, for Fermionic systems, in the Metals/Interacting electrons/Compressibility file).



So say we have some interacting system, with some Fermi surface, and we increase the number of electrons by dn so as to push the Fermi surface outward from kF to F.



What does this do to the chemical potential? Well it goes up. And it’s defined, moreover, as the quasi-particle energy at the new Fermi surface, at least at T = 0, i.e., the energy added to the system when we add a single quasiparticle. For isotropic systems, we know that the quasi-particle energy of one of these added particles is (see previous file):



So the energy at the new Fermi surface will be:



And so the change in chemical potential will just be:



Now have to figure out how this relates to the change in number of particles. Well this is just summing up the number of kσ states between k = kF and k = F. And this is (can be):



So dμ/dn is given by:



And finally we get:



F0s > 0, and so this doesn’t seem to predict any kind of instability, which makes the theory self-consistent at least. Looks like these factors make the compressibility about 1/36 or so that of the free gas. We would expect it to be lower due to the repulsive interaction. Somewhere (Classical Mechanics folder) we once derived a formula for the velocity of sound wave given its bulk-modulus, or compressibility. We found,



We saw in the Free Day/Electrons/Thermal Properties file that this reduced to vs = vF/√3. And so analogously, we’ll have here:



**Magnetic Susceptibility**

Now we’ll add a magnetic field. We’ll ignore, I think, coupling the magnetic field to mobile d.o.f., i.e. the momentum, and just focus on the spin-field interaction. From the previous file, we know the quasi-particle energy levels are given by:



where γ = e/2m (including sign of e), σ = ±1, and H is the external field (technically should be total field, B, but guess we’re presuming these to be about the same, i.e., small induced field). We can directly form an expression for the magnetization:



Now μ depends on T to second order, and can only depend on H to second order too (there is no preferred vector in the system, except for **H**, and so can only depend on **H**·**H** = H2). So we can treat μ as a constant εF\*. And to first order in H, we can do this just like we did in the free case:



(where first term in sum goes to zero because of sum over σ) In the low T limit, we have:



where recall we have established the density of states in the interacting case to simply be ρF\* (m renormalized), but since we’re not summing over spins yet, this is just ρF/2. So what remains is to work out ∂εkσ/∂H. I guess we can do this directly from our equation for εkσ above, and we get:



So now,



And the susceptibility is:



where s = ½.